

THE ASSIGNMENT PROBLEM

Question 1

In an assignment problem to assign jobs to men to minimize the time taken, suppose that one man does not know how to do a particular job, how will you eliminate this allocation from the solution?
(4 Marks)(Nov., 2009)

Answer

In an assignment minimization problem, if one task cannot be assigned to one person, introduce a prohibitively large cost for that allocation, say M, where M has a high the value. Then, while doing the row minimum and column minimum operations, automatically this allocation will get eliminated.

Question 2

A manager was asked to assign tasks to operators (one task per operator only) so as to minimize the time taken. He was given the matrix showing the hours taken by the operators for the tasks.

First, he performed the row minimum operation. Secondly, he did the column minimum operation. Then, he realized that there were 4 tasks and 5 operators. At the third step he introduced the dummy row and continued with his fourth step of drawing lines to cover zeros. He drew 2 vertical lines (under operator III and operator IV) and two horizontal lines (aside task T_4 and dummy task T_5). At step 5, he performed the necessary operation with the uncovered element, since the number of lines was less than the order of the matrix. After this, his matrix appeared as follows:

Operators

Tasks	I	II	III	IV	V
T_1	4	2	5	0	0
T_2	6	3	3	0	3
T_3	4	0	0	0	1
T_4	0	0	5	3	0
T_5 (dummy)	0	0	3	3	0

- (i) What was the matrix after step II ? Based on such matrix, ascertain (ii) and (iii) given below.
- (ii) What was the most difficult task for operators I, II and V ?
- (iii) Who was the most efficient operators?
- (iv) If you are not told anything about the manager's errors, which operator would be denied any task? Why?
- (v) Can the manager go ahead with his assignment to correctly arrive at the optimal assignment, or should he start afresh after introducing the dummy task at the beginning?

(10 Marks)

Answer

	01	02	03	04	05
(given)					
T ₁	4	2	5	0	0
T ₂	6	3	3	0	3
T ₃	4	0	0	0	1
T ₄	0	0	5	3	0
T ₅	0	0	3	3	0
(Dummy)					

Junction values at dummy = 3. 3 was the minimum uncovered element.

Previous step was

7	5	5	0	3
9	6	3	0	6
7	3	0	0	4
0	0	2	0	0
0	0	0	0	0

- (i) At step II the matrix was:

7	5	5	0	3
9	6	3	0	6
7	3	0	0	4
0	0	2	0	0

(ii) For Operator I, Most difficult task will be indicated by hours = T_2

Operator II T_2

Operator V T_2

(iii) Most efficient operator = Operator 4

(iv) If the Manager's mistake was not known,

4	2	5	0	(88)
6	3	3	(88)	3
4	0	(88)	0	1
0	0	5	3	0
0	0	3	3	0

We continue the assignment; $T_1 - O_5$,

$T_2 - O_4$, $T_3 - O_3$ are fixed.

Between T_4 and T_5 , O_1 or O_2 Can be allotted.

So, other O_1 or O_2 Can be denied the job.

(v) Yes, the Manager can go ahead with the optimal assignment

Row minimum is not affected by when the dummy was introduced.

Column minimum was affected. But in the process, more zeros were generated to provide better solution.

Question 3

The cost matrix giving selling costs per unit of a product by salesman A, B, C and D in regions R_1 , R_2 , R_3 and R_4 is given below:

	A	B	C	D
R_1	4	12	16	8
R_2	20	28	32	24
R_3	36	44	48	40
R_4	52	60	64	56

(i) Assign one salesman to one region to minimise the selling cost.

(ii) If the selling price of the product is Rs. 200 per unit and variable cost excluding the selling cost given in the table is Rs. 100 per unit, find the assignment that would maximise the contribution.

(iii) What other conclusion can you make from the above? (8 Marks)(Nov., 2008)

Answer

(i)

4	12	16	8
20	28	32	24
36	44	48	40
52	60	64	56

Subtracting minimum element – each row.

0	8	12	4
0	8	12	4
0	8	12	4
0	8	12	4

Subtracting minimum element – each column,

0	0	0	0
0	0	0	0
0	0	0	0
0	0	0	0

Minimum no. of lines to cover all zeros = 4 = order of matrix. Hence optimal assignment is possible.

Minimum cost = $4 + 28 + 48 + 56 = 136$.

$$= AR_1 + BR_2 + CR_3 + DR_4$$

Since all are zeros, there are 24 solutions to this assignment problem.

Viz.

A	B	C	D
R ₁	R ₂	R ₃	R ₄
R ₂	R ₃	R ₄	R ₁
R ₃	R ₄	R ₁	R ₂
R ₄	R ₁	R ₂	R ₃
R ₁	R ₃	R ₄	R ₂

etc.

A can be assigned in 4 ways, B in 3 ways for each of A's 4 ways.

(ii) $SP - VC = 100$ Rs.

	A	B	C	D
R_1	96	88	84	92
R_2	80	72	68	76
R_3	64	56	52	60
R_4	48	40	36	44

Subtracting the highest term

0	8	12	4
16	24	28	20
32	40	44	36
48	56	60	52

Subtracting minimum term of each row.

0	8	12	4
0	8	12	4
0	8	12	4
0	8	12	4

Which is the same as the earlier matrix

Maximum contribution = Rs. $(96 + 72 + 52 + 44) =$ Rs. 264.

Alternative Solution:

Maximisation of contribution is same as minimizing cost. Hence, same assignments as in (i) will be the optional solution.

Maximum Contribution Rs. $(400 - 136) =$ Rs. 264

- (iii) (a) The relative cost of assigning person i to region r does not change by addition or subtraction of a constant from either a row, or column or all elements of the matrix.
- (b) Minimising cost is the same as maximizing contribution. Hence, the assignment solution will be the same, applying point (i) above.
- (c) Many zero's represent many feasible least cost assignment. Here, all zeros mean maximum permutation of a 4×4 matrix, viz. $4 \times 3 \times 2 \times 1 = 24$ solutions are possible.